CHAMPIONS OF MATHEMATICS

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This book is dedicated
to
William C. Heady

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THE MATHEMATICS OF MUSIC

Pythagoras was one of the first great mathematicians whose name is known. He lived about 2,500 years ago, and he was born on the island of Sámos in the Aegean Sea. Sámos is the nearest island to Turkey on the mainland. During Pythagoras' day, it was one of the leading Greek cities and a successful shipping center. It had a good trade with Egypt and Libya in Northern Africa and with Corinth and other cities in Asia Minor.

Pythagoras' father was a merchant, originally from Tyre (a Phoencian city now part of Lebanon). Tyre was frequently mentioned in the Bible. Hiram, king of Tyre, furnished building materials for Solomon's Temple (1 Kings 9:11). This was about 500 years before the time of Pythagoras. Jesus visited the area around Tyre (Mark 7:24). This was about 500 years after Pythagoras lived.

Pythagoras traveled with his father who bought and sold grain. Their journeys took them as far north as Italy and as far south as Syria. Few other people visited so many interesting places.

He enjoyed playing the lyre and reading poetry. He memorized parts of the great heroic tales by Homer about events surrounding the Trojan wars. Pythagoras, like so many before and after, was thrilled by the exploits of Achilles, the greatest of the Greek warriors. He read about the Greek gift of the wooden Trojan horse, filled with soldiers, that gained entry inside the walled city of Troy. He followed the trials of brave Odesseus in his wandering and when he regained his home after the war.

Pythagoras had an interest in learning. He coined the word "philosophy," which means love of knowledge. Until recently, scientists were known as natural philosophers. A natural philosopher was a person who attempted to understand the natural world. Not only was Pythagoras one of the first scientists, but he coined the name by which they were known.

He visited Miletus, a rival city to Samos, and there met the legendary Thales, the first of the Greek scientists. In astronomy, Thales predicted an eclipse of the sun a year before the event. The eclipse took place on May 28, 585 B.C. The date is known so well because the frightful spectacle halted two armies (the Medes and Lydians) as they advanced to war.

Thales also showed a simple way to measure the height of tall buildings. He took a stick of known length and measured the length of its shadow. Then he compared its length to the length of the shadow of the building. The ratio of the one to the other gave the missing height. Thales measured the height of the pyramids in this way. He measured the length of the shadow of the Great Pyramid of Giza, and it was 214 cubits long. A measuring rod that is six cubits high casts a shadow of four cubits. A simple calculation of the proportion (pyramid height / length of pyramid shadow = rod height / length of rod shadow = pyramid height / 214 = 6/4) gave the height of the pyramid as 321 cubits. Multiply 214 by 6 and divide by 4. A cubit is 18 inches, so the pyramid was a little more than 481 feet high.

This idea of ratios and proportion impressed Pythagoras. He began applying numbers to other areas of science. Thales advised him to study in Egypt. Pythagoras traveled to Egypt. He was taken prisoner when Persia invaded that country. He was carried off to Babylon (now modern day Baghdad in Iraq). Pythagoras took the opportunity to learn more about astronomy from his captors. After five years he gained his freedom and returned home.

About 530 B.C., he settled in Crotona (now Crotone), a Greek colony in southern Italy. He founded a school and tested his ideas about how people should conduct their lives. He wore simple clothes and avoided a show of wealth. He encouraged students to examine themselves and identify their thoughts and emotions.

Pythagoras may have written books, but none of them survive today. His students took notes of his lectures and added to them. It is sometimes difficult to separate his ideas from those of his students.

Scientists do know that he emphasized the role of numbers in describing the natural world. He studied music and sound. His discoveries are surprisingly accurate and still true today.

In almost all of the other sciences — astronomy, physics, biology, chemistry, and medicine — the ideas expressed by the ancient Greeks have proven to be incomplete or in error.

The exception is mathematics. The ideas of Pythagoras and the other Greek mathematics such as Euclid have endured to this present time. Pythagoras was not only correct in mathematics, but also his study of numbers as the foundation of musical tones has proven to be correct, too.

Pythagoras noted that musical tones were more pleasing when the strings vibrated in sections that were ratios of whole numbers. A string of a lyre produced sound because of the vibration of the string. Suppose the entire string vibrated. Then it produced a low tone known as the fundamental pitch. If it were touched in the middle, the string would vibrate in two parts. The pitch would be twice as great as the fundamental. It was the first overtone. If the string were touched near the top, it would vibrate in three parts and produce the second overtone. If two strings vibrated in simple multiples of one another such as 2 to 3, this also gave a pleasing tone. His study of the mathematics of music is one branch of science that has remained unchanged into modern times.

Pythagoras was a good musician, and would play for students who were ill, to improve their spirits. People of that time often turned to music to soothe their troubled minds. The Bible tells that David in the Old Testament was a musician who played for King Saul. "David would take his harp and play. Then relief would come to Saul; he would feel better" (1 Sam. 16:23).

Pythagoras was fascinated by numbers and identified their special properties. To him, each number had a personality of its own. He arranged pebbles to represent numbers as triangles, squares, or other shapes.

The number three he visualized as a triangle of pebbles. Three can be shown as a single pebble on top and then a row of two pebbles. Ten was a triangle of pebbles. One pebble on top, two in the second row, three in the third row and four in the final row: $1 + 2 + 3 + 4 = 10$.

These 15th century Italian woodcuts imagine Pythagoras proving his ideas of harmony by various tests on bells and on glasses of water (top), on string tensions (middle), and on lengths of columns of air (bottom).

Four was a square number. It could be written as two rows of two pebbles in the shape of a square. Nine was the next square, three rows of three pebbles.

Some numbers were prime, such as seven, and could not be divided by any other number except one. Others were composite, such as 12, which had many divisors. He called 28 a perfect number because it was the sum of its divisors: 1 + 2 + 4 + 7 + 14 = 28.

His best-known achievement was the proof of what is today called the Pythagorean theorem. A theorem is a statement that can be proven mathematically correct. The Pythagorean theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

Many ancient people had learned this simple rule relating the three sides of a right triangle. A right triangle is one that has one angle of 90 degrees. The two lines connected to the right angle are the legs. The side opposite the 90-degree angle is the hypotenuse. Ancient engineers knew that the sum of the squares of the length of the legs equaled the square of the hypotenuse. For instance, suppose a right triangle has legs of lengths three and four. The hypotenuse will have a length of five because $3^2 + 4^2 = 5^2$. The superscript 2 means to multiply the number by itself: 3 * 3 + 4 * 4 = 9 + 16 = 25.

The Egyptians made a practical application of the Pythagorean theorem by taking a cord and knotting it at 12 equally spaced intervals. They held the cord in the shape of a triangle so that one side had three knots and the other side four knots. The last side would have five knots and the angle opposite that side would be a right angle. It helped them lay out their buildings with square corners. The use of the knotted cord was a successful rule of thumb discovered by trial and error.

Pythagoras saw a difference between trial and error and true scientific proof. Pythagoras gave a proof of the Pythagorean theorem. When Pythagoras lived, the notation for showing numbers that we use today had not been invented. Instead, letters of the alphabet stood for numbers. The Greek letter delta, D, represented 10, the Greek letter H represented 100, and the Greek letter chi V represented 1,000. This was a clumsy system and one that was difficult to use.

Rather than proving the Pythagorean theorem with numbers, Pythagoras used geometric figures. He constructed two squares with sides the same lengths as the two sides of the triangle. He showed that their combined area (enclosed space) would equal the area of a square with sides the length of the hypotenuse.

Pythagoras believed that everything could be represented as the ratio of whole numbers. For instance, the pleasing tones of musical instruments were ratios of 1 to 2, 1 to 3, or some other ratio of simple whole numbers. The ratio of the legs of the right triangle that the Egyptians used was 3 to 4.

He believed the common whole numbers (1, 2, 3, 4, and so on) and the fractions formed by them (1/2, 1/3, 2/3, 1/4, 3/4, and so on) could measure all quantities in mathematics and nature. Numbers formed by the ratio of whole numbers are called rational numbers. Pythagoras believed all numbers were rational.

He proved himself wrong while studying the diagonal of a square. The sides of a square are the same lengths. The diagonal of a square is a straight line that connects one corner of the square to the opposite corner and goes through the center of the square. What is the length of the diagonal compared to the sides of the square?

The Pythagorean theorem gives the answer. The diagonal separates the square into two identical right triangles. The legs of the triangle are sides of the square and the hypotenuse of the triangle is the diagonal of the square.

The Pythagorean triangle gives the length of the diagonal by the equation $\text{side}^2 + \text{side}^2 = \text{diagonal}^2$. Suppose the sides of the square have a length of one. When one is squared (multiplied by itself) the answer is one: $1^2 = 1 * 1 = 1$. The length of the diagonal, represented by x, is given by $1^2 + 1^2 = x^2$, or $1 + 1 = x^2$, or $2 = x^2$.

Pythagoras tried to determine what ratio for x, when multiplied by itself, would give 2. The number 10/7 is close because $(10/7)^2 = 100/49$ or about 2.04. That is close but not exact. In fact, no ratio, when multiplied by itself will give two exactly. Rather than being a rational number, the square root of 2 is an irrational number. Irrational means without a ratio. No ratio or decimal fraction can equal it exactly.

This unexpected discovery of irrational numbers was so astonishing that Pythagoras and his students kept it secret for many years. Eventually, the proof was made public and other irrational numbers were discovered. For instance, the ratio of the distance around a circle (circumference) divided by the distance through the center of the circle (diameter) is irrational. It is represented by the Greek letter pi, π. It can be approximated by the ratio 22/7 (about 3.14), but like the square root of 2, pi cannot be given exactly by the ratio of two whole numbers.

In astronomy, Pythagoras stated that the earth was spherical and not flat. He realized that the morning and evening star were the same body (Venus.) He thought numbers controlled the crystalline spheres that gave planets their motions. He tried to relate this motion to the simple ratios of music. The planets do follow simple numeric laws, but not those from music. Pythagoras' idea of music of the spheres put astronomers on the wrong track for many years.

Pythagoras had selected the Greek city of Crotona for his school to avoid political turmoil and to be safe from advancing armies and wars. However, he and his students with their secrecy, unusual dress, and different ways were not entirely accepted by the local inhabitants. They were viewed as a cult and came under persecution. The last days of Pythagoras are unclear. Many scholars believe he fled to Megapontum, a Greek city in southern Italy, where he died.

Exactly how he died is not clear. What is clear is that he made two important advances in mathematics. First, Pythagoras set the standard that mathematical results must be based on mathematical proof, not rules of thumb or vague ideas of what seems reasonable.

Second, when he thought his understanding of mathematics was complete, he discovered irrational numbers. The Bible, in Job chapters 38 and 39, contains a list of questions to show the richness of God's creation and the challenge of learning about the physical world. Time and again, mathematicians and scientists would think they had completely revealed a subject only to be proven wrong by a new discovery. Pythagoras was one of the first to learn this important lesson.

Pythagoras also noticed the harmony in nature. Everything fit together with a unity in the design. Great achievers in science and mathematics have noticed this fact over and over. Great mathematicians who came later such as Isaac Newton and Carl Gauss would recognize the design as being put there by the Creator. Pythagoras was a pagan who did not have access to the Word of God. But even in his ignorance, he saw the hand of the Creator in his studies.